Design of an ultimate storage ring with 10 pm emittance

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5/6/2011

Outline

- Purpose and background for 10 pm storage ring.
- TME structure and combined function magnets lattice.
- Driving terms and dynamic aperture optimization.
- Intrabeam scattering and its effect on beam properties.
- Microwave instability and possible FEL process study.
- Conclusion.

What is 10 pm?

- Natural emittance with 10 picometer(pm) in both planes would greatly enhance the brightness simply due to the decrease in transverse beam size.

$$\varepsilon_n = \beta_r \gamma_r \varepsilon_{un} \qquad \sigma_y = \sqrt{\beta \varepsilon_{un}} \qquad \sigma_{y'} = \sqrt{\gamma \varepsilon_{un}}$$

$$B \propto \frac{N_e}{\varepsilon_x \varepsilon_y \varepsilon_z}$$

- When beam has such a small emittance, it reaches the diffractive limit.

$$\varepsilon_u \approx \frac{\lambda}{4\pi}$$

for hard x-ray 1 Å, $\varepsilon_x \approx 10^{-11}$m.

Transversely coherent!

Coherent radiation

- If we have a transversely coherent radiation, the intensity of radiation would be further more enhanced.

Non-coherent

$$E_r^{\ 2} \propto N_e$$

Coherent

$$E_r^{\ 2} \propto N_e^{\ 2}$$

- Mode number M_T

$$M_T = \frac{\sigma_x \sigma_{x'}}{\lambda / 4\pi}$$

Indicates how many transverse modes the beam carries– how many peaks when Young's interference pattern happens

Why 10 pm not Linac?

- When talking about hard x-ray radiation, Linac-based SASE seems to be a natural choice due to the high gain and lack of suitable oscillator. It has advantages in the following aspects:
  - Narrow bandwidth radiation with tunable central frequency.
  - High peak brightness~ 10-13 orders higher than storage rings.
- Nowadays many storage rings(PETRA III, NSLS II, MAX IV…) have been upgraded or planned to upgrade to have nm or sub-nm transverse emittance with strong damping wigglers. Comparing to Linac-based SASE FEL, rings are strong at:
  - Wide, easily-tunable spectrum from IR to gamma-rays.
  - High average flux due to the high rep rate and high average brightness.
  - Great stability in beam size/shape and positions. Noise and variation from shot to shot is very low.

How to achieve 10 pm?

- Theoretically, the limit of emittance is given by

$$\varepsilon_{un} = F C_q \gamma^2 \theta^3 \quad \text{with} \quad C_q = 3.83 \times 10^{-13} m \quad \text{and} \quad F \propto \langle H \rangle_{dip}$$

- We choose 5GeV beam, thus bending angle of each dipole must be very small.

$\langle H \rangle_{dip} = \gamma D^2 + 2\alpha D D' + \beta D'^2$ minimized → lattice is matched

However, there is a limit for different type of lattice when doing this minimization.
When this minimum is achieved or approached, we call we have achieved TME– theoretical minimum emittance.

How to achieve TME

- The goal is to make <H>$_{dip}$ minimum. Starting from transfer matrix in dipole, we can write dispersion functions and H-function in dipole as such

$$D = \rho(1-\cos\phi) + D_0\cos\phi + \rho D_0'\sin\phi$$
$$D' = (1-\frac{D_0}{\rho})\sin\phi + D_0'\cos\phi$$

$$H(\phi) = \gamma D^2 + 2\alpha D D' + \beta D'^2 = H_0 + 2(\alpha_0 D_0 + \beta_0 D_0')\sin\phi$$
$$-2(\gamma_0 D_0 + \alpha_0 D_0')\rho(1-\cos\phi) + \beta_0\sin^2\phi + \gamma_0\rho^2(1-\cos\phi)^2$$
$$-2\alpha_0\rho\sin\phi(1-\cos\phi)$$

- Depending on the lattice(whether acromat or not), different minimization process is used.

Emittance minimization

- Choose the scheme for non-acromat case, calculate $\langle H \rangle_{dip}$ by averaging the H over all phase across the dipole and calculate extremum:

$$\frac{\partial \langle H \rangle_{dip}}{\partial D_0} = \frac{\partial \langle H \rangle_{dip}}{\partial D_0'} = 0$$

Using small angle approximation

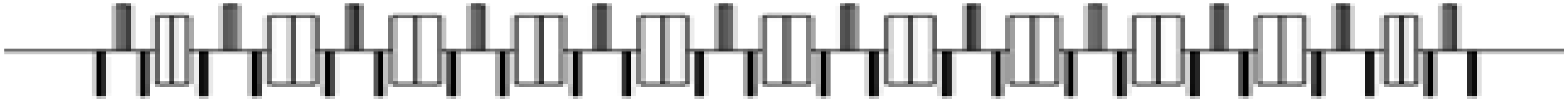
$$D_0^* = \frac{1}{6} L\theta$$

$$\beta_0^* = \frac{L}{\sqrt{60}}$$

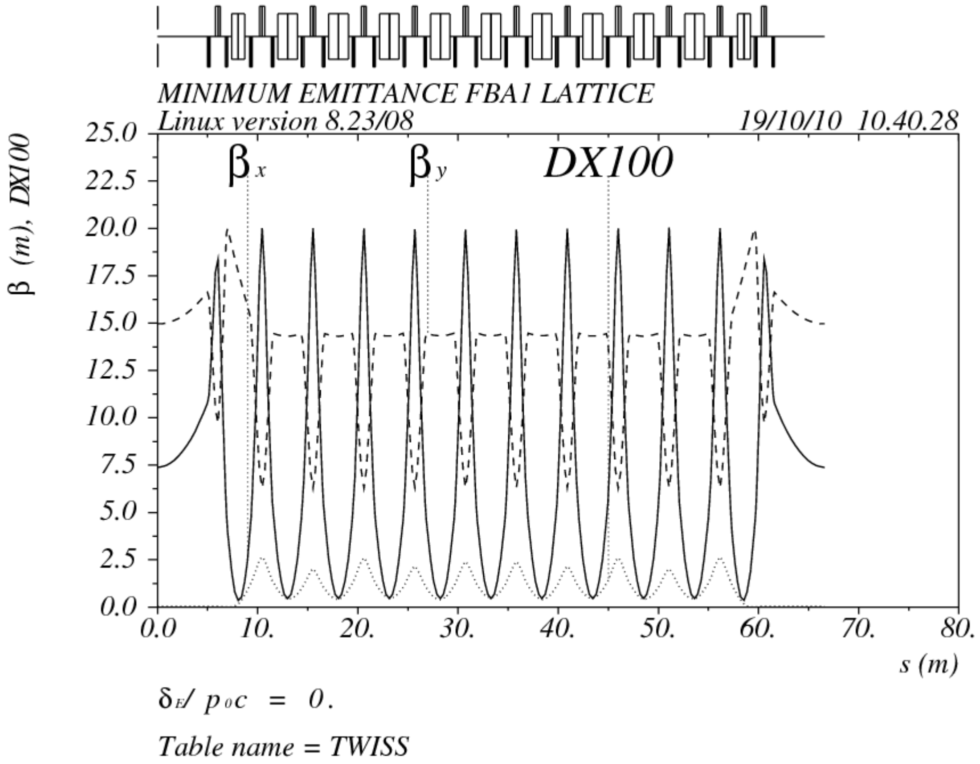
The minimum of beta function and dispersion happen at the center of dipoles

10 pm storage ring

- 10pm storage ring has 40 superperiods with 11 bending magnets in each superperiod.
- Center 9 dipole cells have non-zero dispersion and quadrupole triplets are used to match the dispersion and beta functions to theoretical miminum.
- Outer 2 dipole cells are used to match the dispersion thus we have 10m long non-dispersive straight sections between superperiods for insertion.
- The whole ring can be designed into race track shape and two long straight sections (100m) can be used for FEL process.



10 pm storage ring(cont'd)



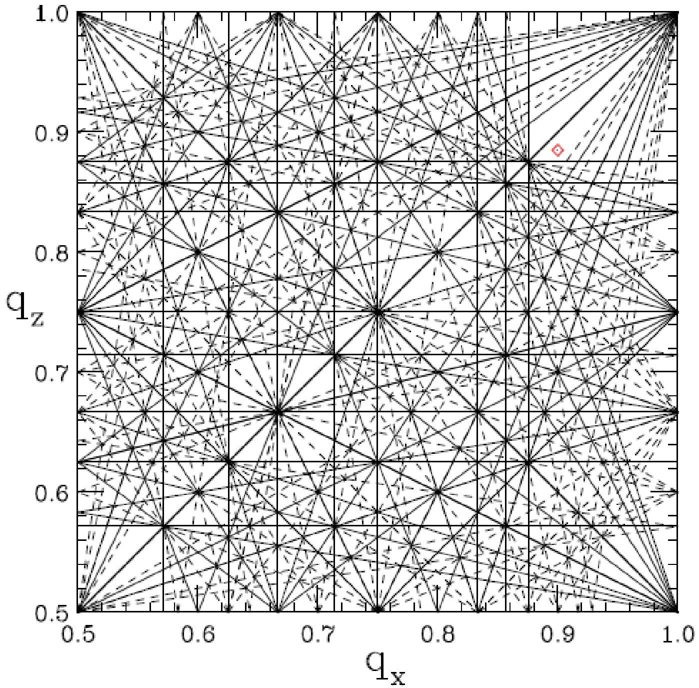
Twiss parameters for one superperiod with dispersion function magnified by 100 times.

Parameters	Value
Circumference	2663m
Energy	5GeV(4-7GeV)
Natural chromaticities	-595.339(horizontal) -148.741(vertical)
Qx	202.9
Qy	33.884
dE/E	3.8e-4
Momentum compaction factor	1.223e-5
Natural emittance	9.1pm(before coupling)

Large natural chromaticities are induced by large number of dipoles and small beta functions.

10 pm storage ring(cont'd)

- By tuning the quadrupole triplet located in the non-dispersive region, we can move the betatron tunes to a spot far from lower order resonances.



Tune space with up to 8th order resonance lines. Red square shows the tunes for 10 pm storage ring.

Strategy in shorten circumference(1)

- The periodic structure of this design is just like toy bricks. We can choose # of dipoles in each superperiod and # of superperiods rather freely without changing the optics as long as we keep dipole's length and bending angle the same.
- Considering using a new lattice with 23 central dipoles. So we choose 17 superperiods instead of 40 to keep the bending angle of the central dipoles the same.

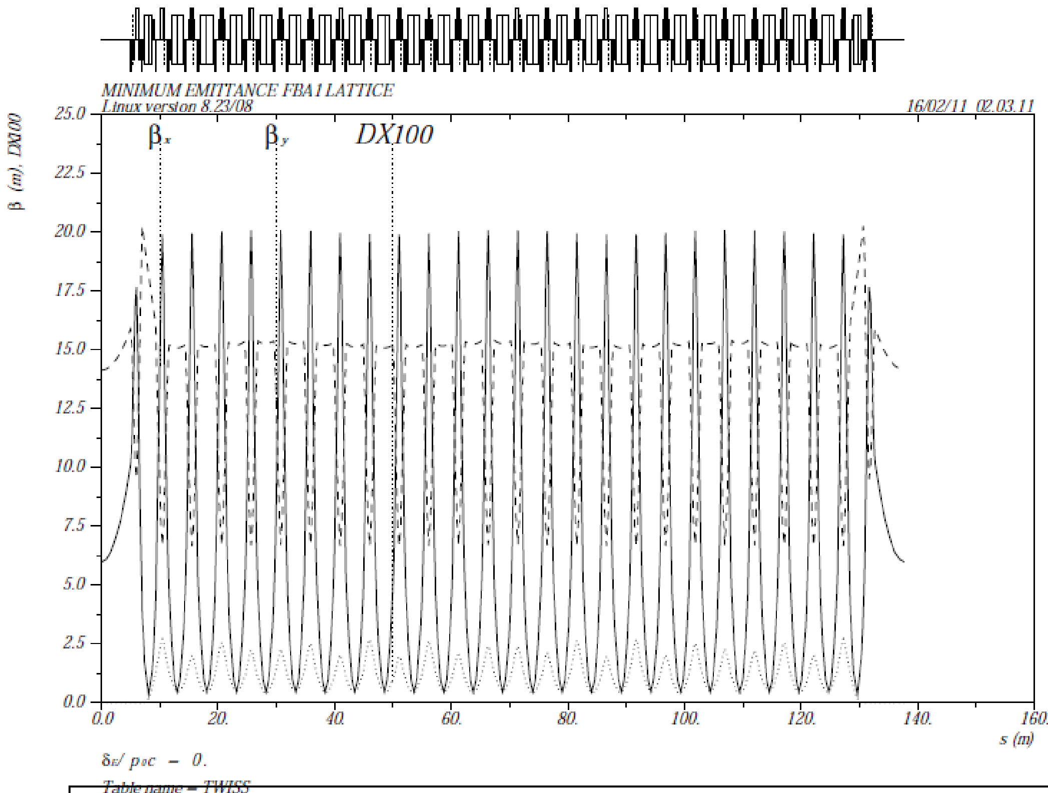
$$\frac{2\pi}{17(1.5*23+2)}*1.5 \approx 0.015189 \approx 0.01512 \approx \frac{2\pi}{40(1.5*9+2)}*1.5$$

New lattice-25BA

Central dipole bending angle

Old lattice-11BA

Result and performance

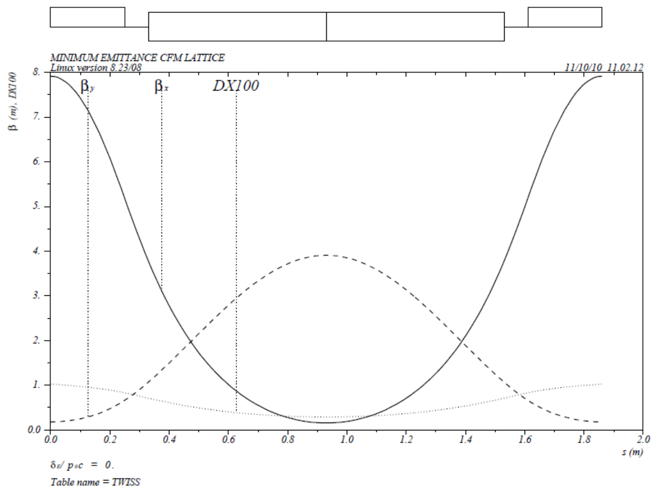


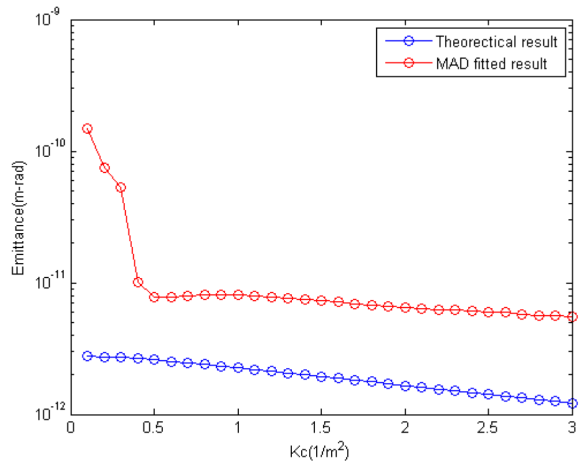
Parameters	Value
Circumference	2334m
Energy	5GeV
Natural chromaticities	-585.427(horizontal) -141.382(vertical)
Qx	189.9
Qy	22.884
dE/E	3.78e-4
Momentum compaction factor	1.4e-5
Natural emittance	9.5pm

Has exactly the same optics and emittance but shorter circumference. Note that this won't improve DA. Natural chromaticities are not changed much due to similar total number of dipoles and ratio in betax/betay.

Strategy in shorten circumference(2)

- Using combined function magnets, we can minimize the number of magnets for optics matching. Instead of using quadrupole triplet, we use singlet while making the dipole with gradient.
- After some data analysis to match the lattice, we find out





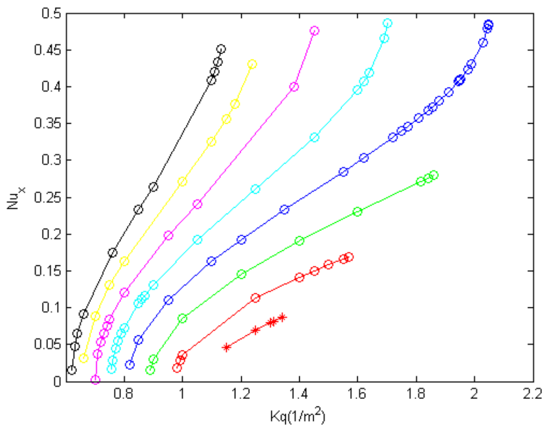
A factor of 4 discrepancy is induced by imperfection of the matching model for combined function dipoles in MAD8.

Sort of "analytical" way

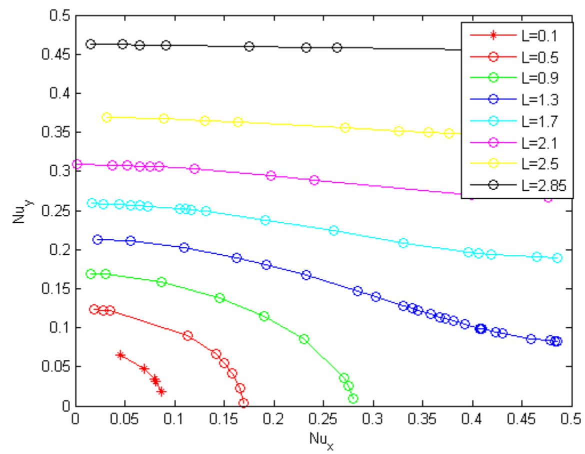
- In this process, the central dipoles' length and bending angle are fixed to provide a predetermined rms emittance and rms energy spread.
- No matching process is carried out in MAD8 and we manually change all the parameters(drift length– L; dipole gradient– Kc; matching quadrupole Kq) to search for best solution.
- We choose one Kc and then vary L and Kq to calculate beta function, dispersion and betatron tunes if the periodic solution exists. And then change Kc and do this again. Each Kc would have a set of band plot with different L's and Kq's.

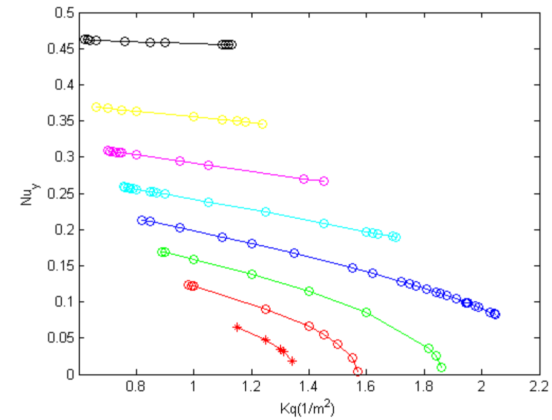
Phase stability diagram

- For a fixed Kc, by varying drift space length, we obtain Necktie diagram.



The boundary reaches stability limit.

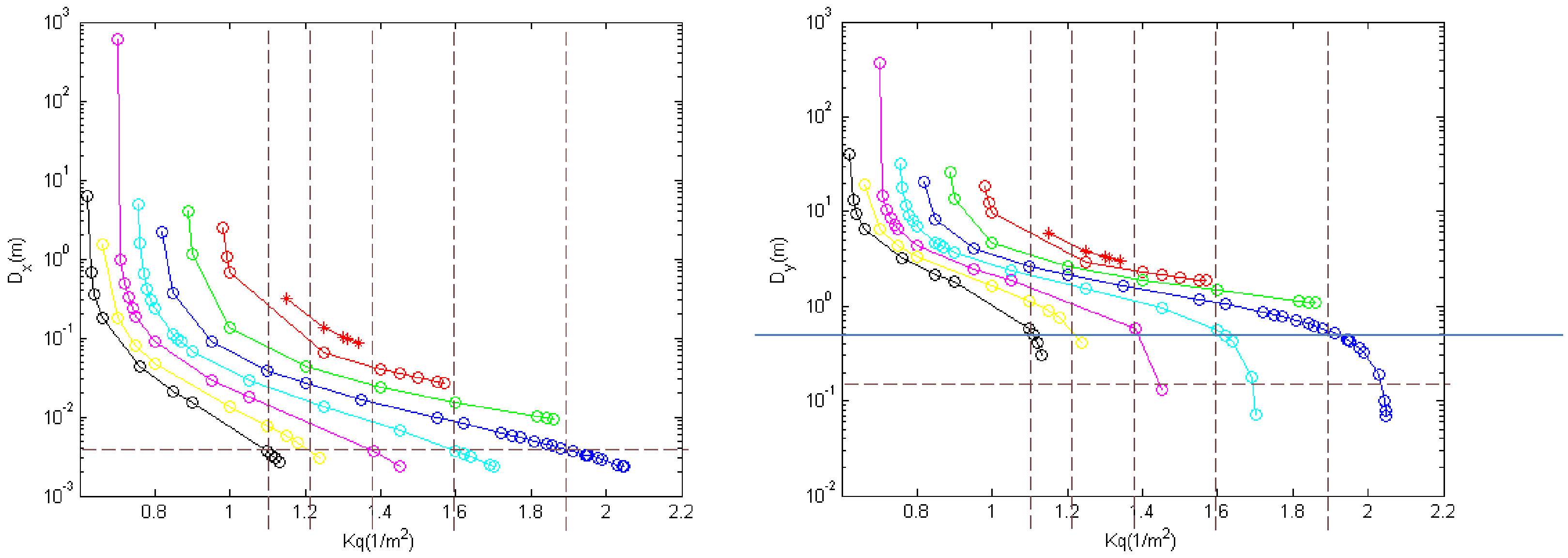




Almost cover phase region from 0 to Pi. Smaller betay function has greater effect in changing tune.

Dispersion and beta function

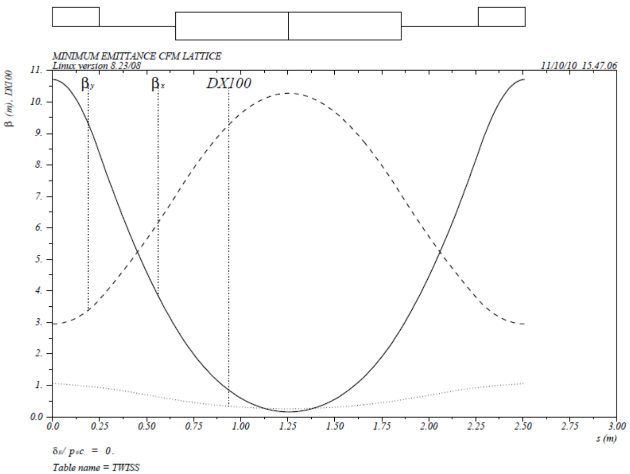
- Similarly, we have dispersion and beta function curves. The dashed lines indicate the theoretical result.



No solution for this case, Kc=0.5 is too small.

First solution

- As we gradually increase the Kc value to about 1.0. First solution shows up with beta function matched to 0.15 m and dispersion 3e-3 m. Cell length is also nice compact– only 2.5 m. Emittance is 6.8 pm with 440 dipoles. Total circumference is 1100 m.

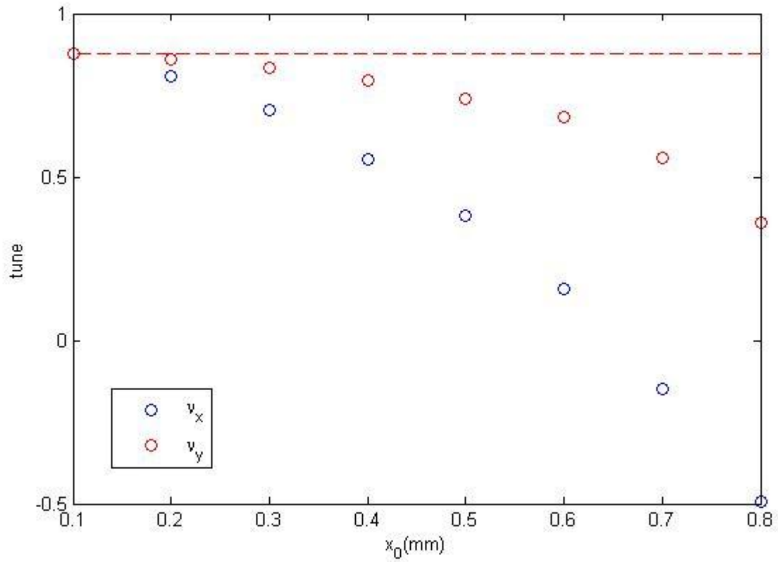


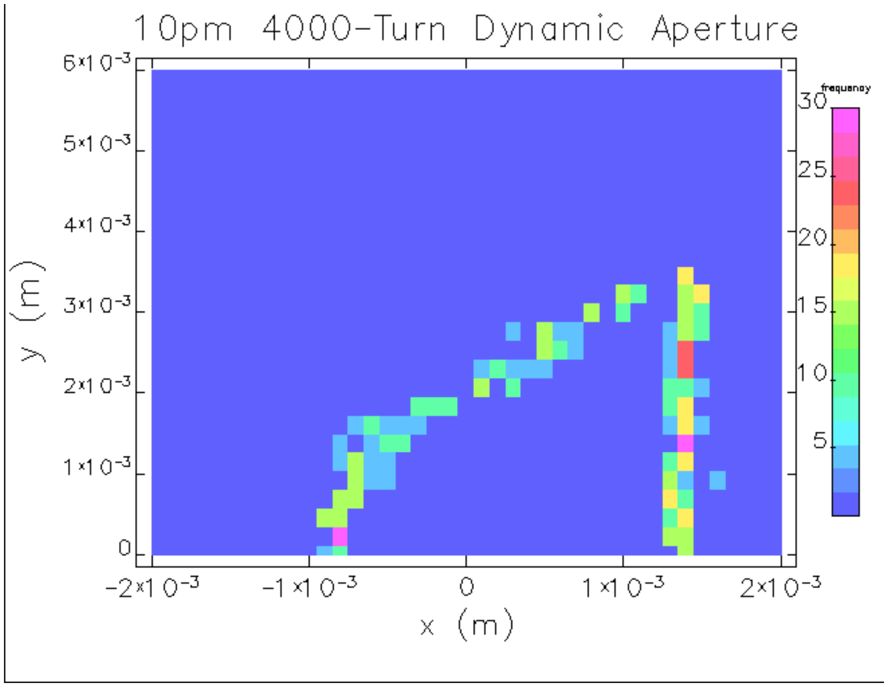
Calculated $B_1/B = Kc*\rho \approx 78 m^{-1}$, not possible for magnet fabrication. It is almost impossible to make a very large dipole with high gradient!

Dynamic aperture and driving terms optimization

- Beam properties listed before are equilibrium values and it takes usually few thousand turns for the beam to damp to that point. Study of dynamical aperture and nonlinear driving terms is crucial point for the design.
- Usually driving term h_{abcde} drives certain order of resonances or quantities. E.g. h_{30000} drives $3\nu_x$ resonance and h_{10020} drives ν_x-$2\nu_y$ resonance.
- After careful study, it turned out that for this lattice design, the tune shift with amplitude, $d\nu/dJ$ is most influential and thus limits the dynamic aperture size.

Dynamic aperture and driving terms optimization(cont'd)





Beam crosses many resonances even with small offset due to the large tune shift with amplitude.

DA after correcting dv/dJ, comparable to most of the USR designs– gives about 60 times of rms beam width.

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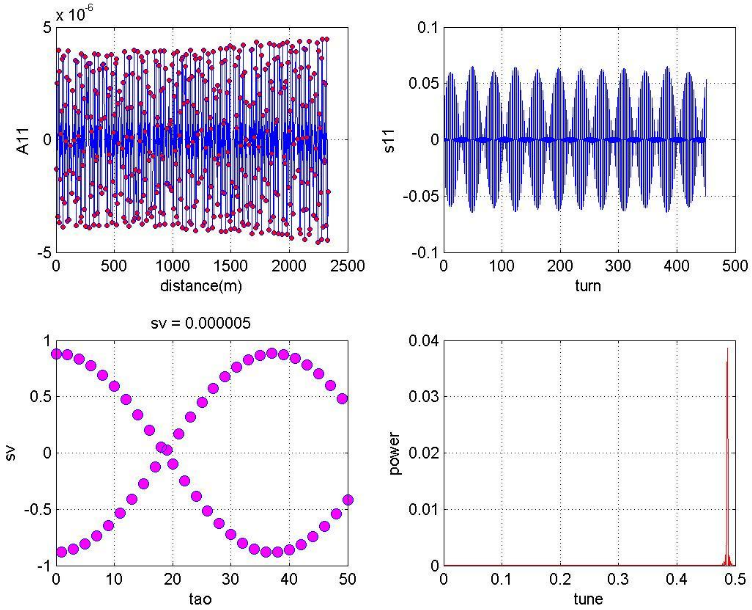
Resonances at DA boundary

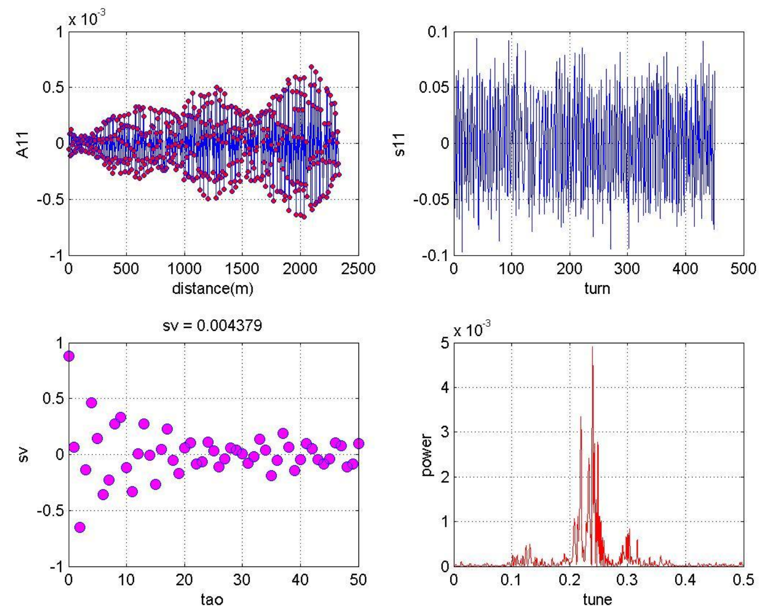
- We study the resonances at DA boundary using Independent Component Analysis(ICA) method. A chaotic pattern shows up indicating there are many resonances overlapping at the DA boundary and particle motion will be chaotic but still bounded.

- Eigenmodes with small amplitude given by ICA shows a much cleaner frequency spectrum.

- This method can be further used for driving terms optimization to improve DA.

Resonances at DA boundary(cont'd)



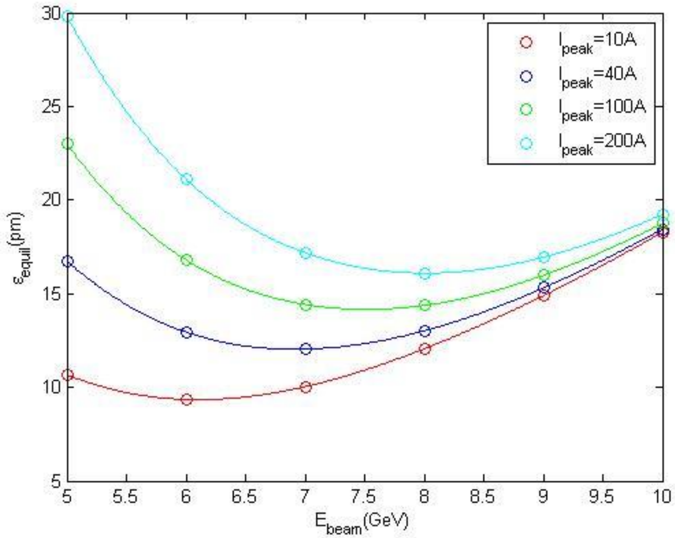


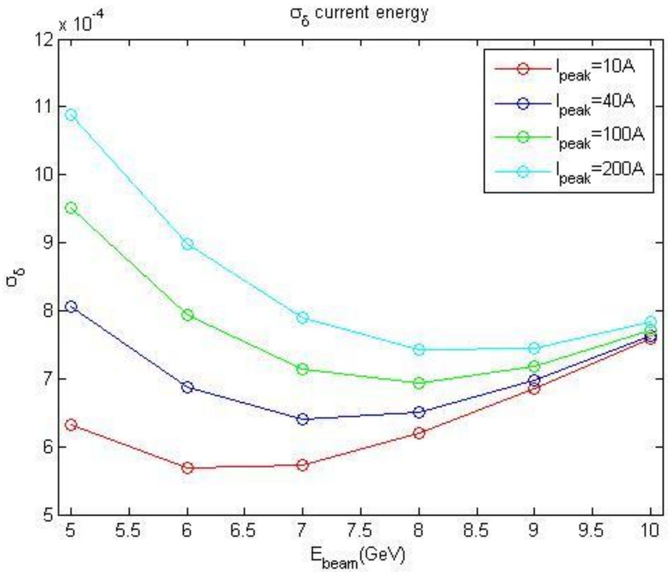
A typical ICA mode with x_0=0.1mm. Spatial and temporal wave function indicates this is a betatron motion mode with nice periodic correlation function. Frequency spectrum indicates this is a 3rd order betatron mode.	The same ICA mode with x_0=1.3mm. Non-periodicity in spatial and temporal wave function indicates instability and correlation function damps to zero gradually over turns– noise like pattern. Noisy frequency spectrum shows many resonances at boundary.

Intrabeam scattering

- IBS(intra-beam scattering) is short-range coulomb scattering which is a diffusive process.
- It is a serious problem when the peak current is high and the transverse emittance is small.
- Similar to space charge effect, it will deteriorate beam quality and blow up horizontal emittances and energy spread which causes serious beam loss.
- Radiation damping, quantum excitation and IBS will reach an equilibrium state and beam coupling plays a key role in this process.

Intrabeam scattering(cont'd)





Same lattice is used for all cases so emittance scales with energy to square power when IBS effect is not significant.

Emittance and energy spread can grow up a few times by the IBS effect. Beam coupling is assumed to be 100%.

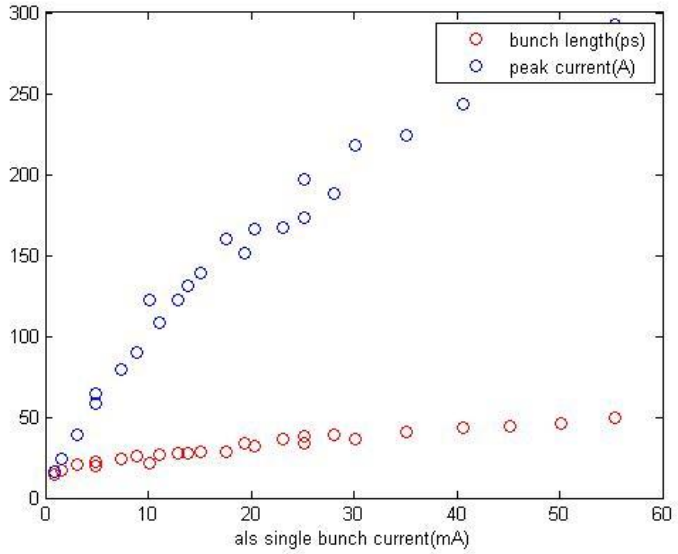
Microwave instability

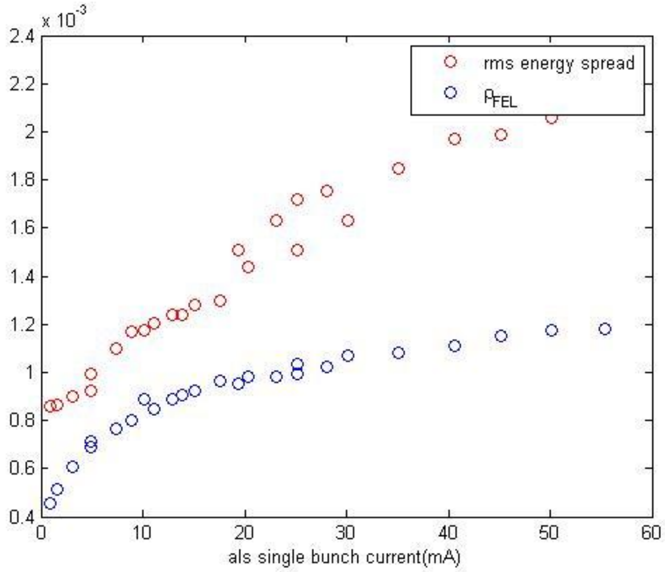
- The non-smoothness of chamber wall or other broadband impedances will generate wake field. This wake field can affect the particles passing thru and sometimes this effect is collective and cause beam emittance, energy spread and bunch length increase thus causes beam loss.

- When collective instability happens, it deteriorates beam qualities very fast and damping does not have time to react.

- Usually in light sources, single bunch beam properties are largely limited by microwave instability.

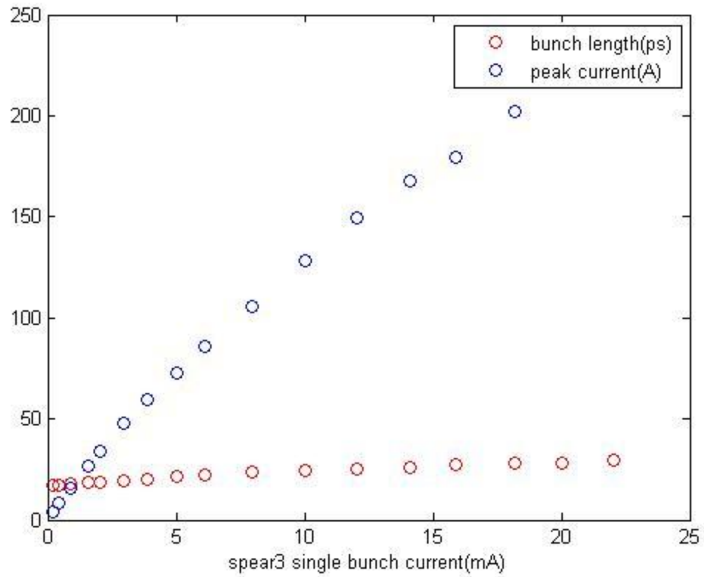
Microwave instability(cont'd)

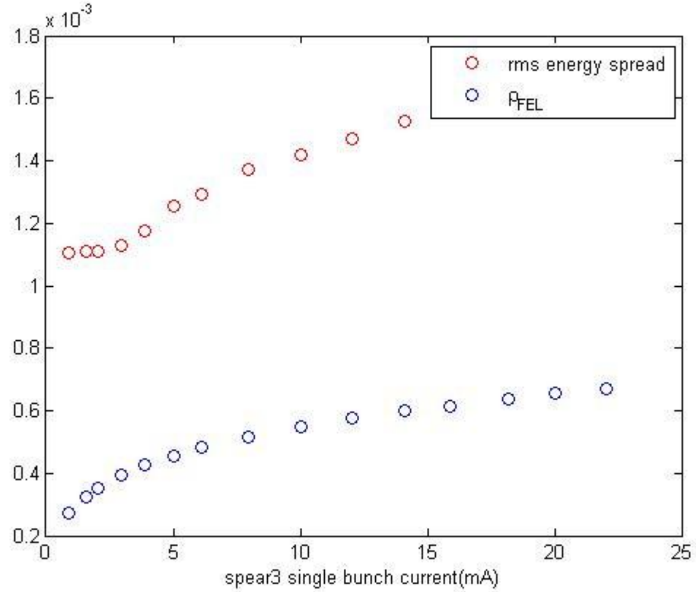




A typical 3rd generation light source– take ALS as example, has energy spread ~10^{-3}, and alpha$_c$~0.001.

Microwave instability(cont'd)



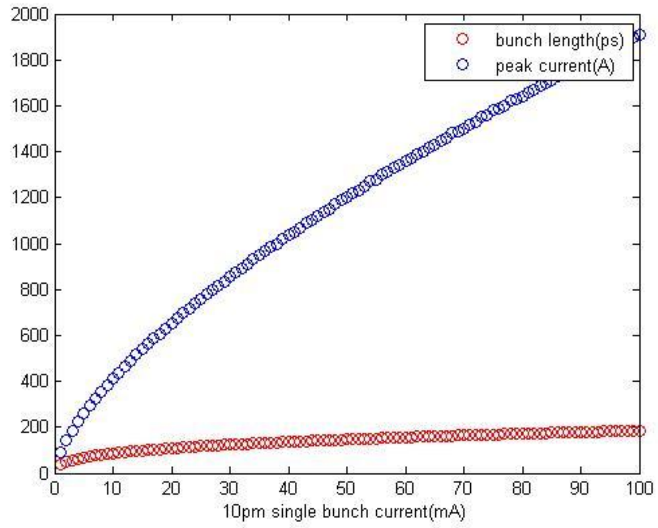


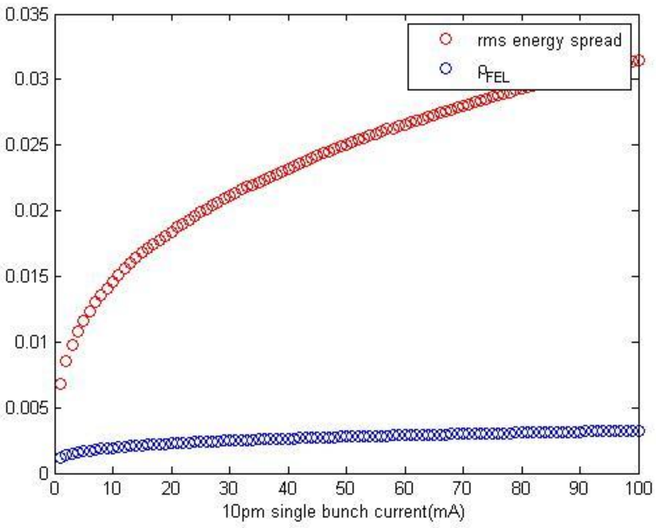
Another example- SPEAR3 has similar beam performance as ALS due to the similar size of the ring ~200m and energy spread ~10^{-3}, and alpha$_c$~0.001.

Microwave instability(cont'd)

- For 10pm storage ring, energy spread is greatly increased by low alpha_c(big size of the ring). Thus ρ_{FEL} is much smaller than σ_E although peak current can be high.

$$I_{thres} = \frac{\sqrt{2\pi}\beta^2 (E/e)\eta^2 \sigma_\delta^3}{|Z_{//}/n|\nu_s} = 7\times10^{-4} mA$$

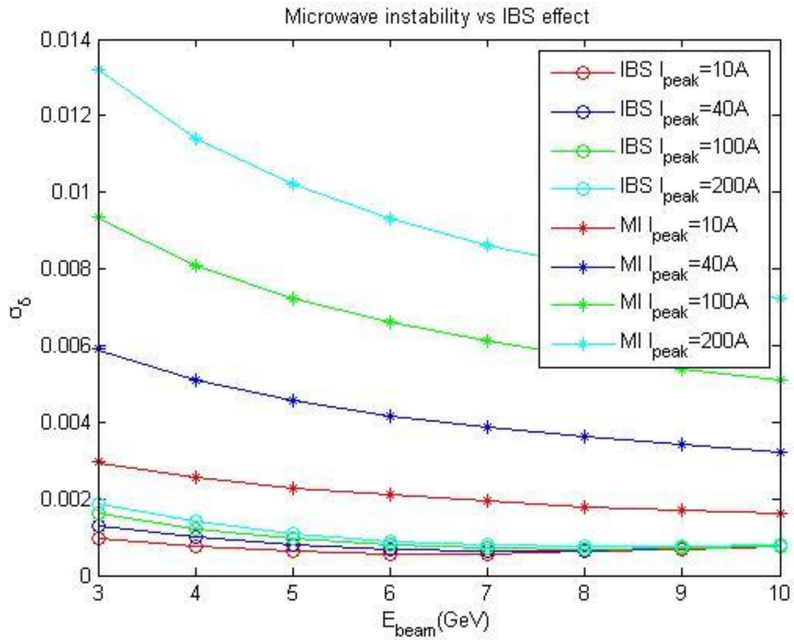




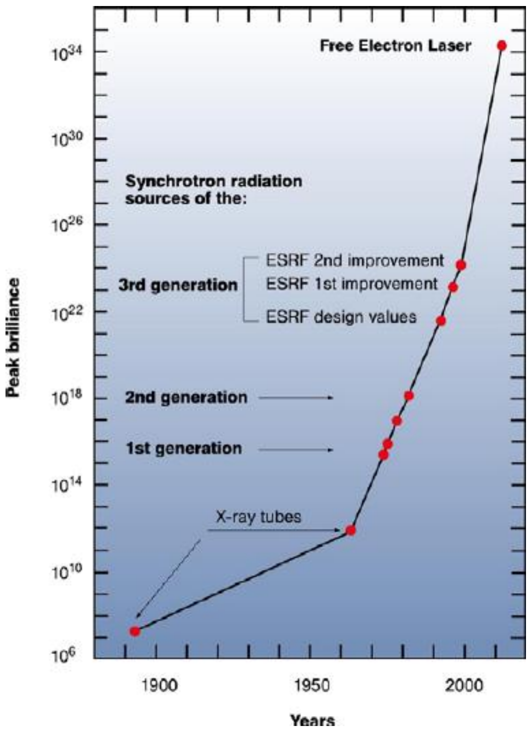
Longitudinal impedance $|Z_{||}/n|$=0.5 ohms for this calculation, although lower value has been proven possible in reality. This value is chosen to show the effect more clearly.

IBS VS Microwave instability

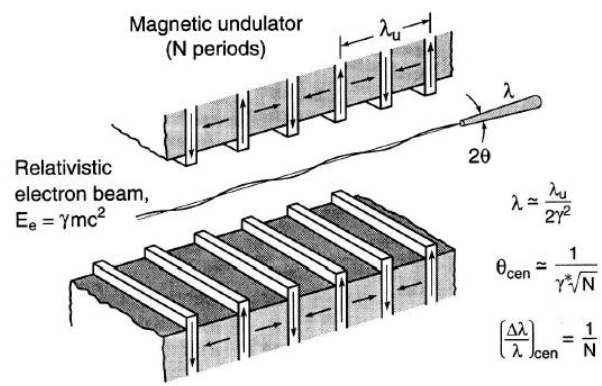
- They are caused by different mechanisms but they are both bad for beam properties and ring operation.
- Microwave instability has much bigger effect than IBS and should be carefully taken care of.



Undulator and free e- laser



Undulator is composed of alternating magnetic field, particles in the field will perform wiggling motion and lose or gain energy from the field depending on the phase. The periodic structure can also be used to generate temporal coherent radiation which has much higher brilliance than ever.



Oscillator FEL

- Oscillator FEL uses a combination of low current electron beam and an optical cavity to trap the optical pulse in several passes through the wiggler. Each pass involves a new electron bunch interacting with the optical field and undergoing microbunching at the end of the wiggler. The optical intensity grows by a small amount (low gain) in each pass with an amplification factor of $1 + g_{ss}$.

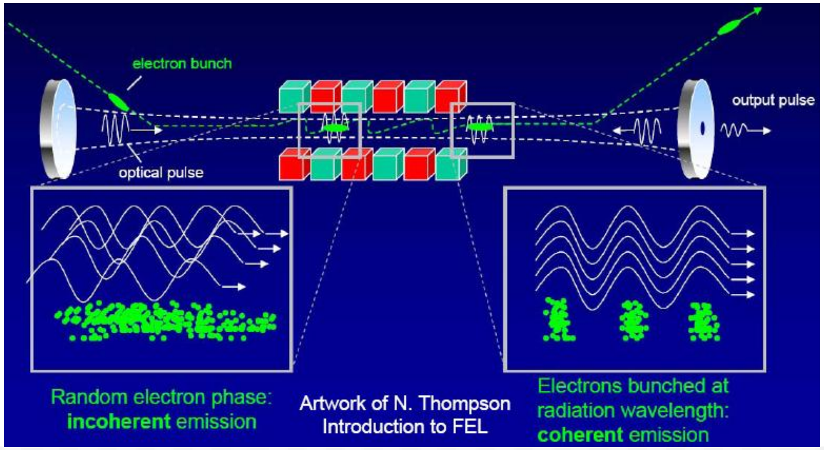
Perfect for rings with lower peak current comparing with LINAC

Low gain thus reaching saturation slower and needs fine adjustment of high quality mirrors

Lasing scheme

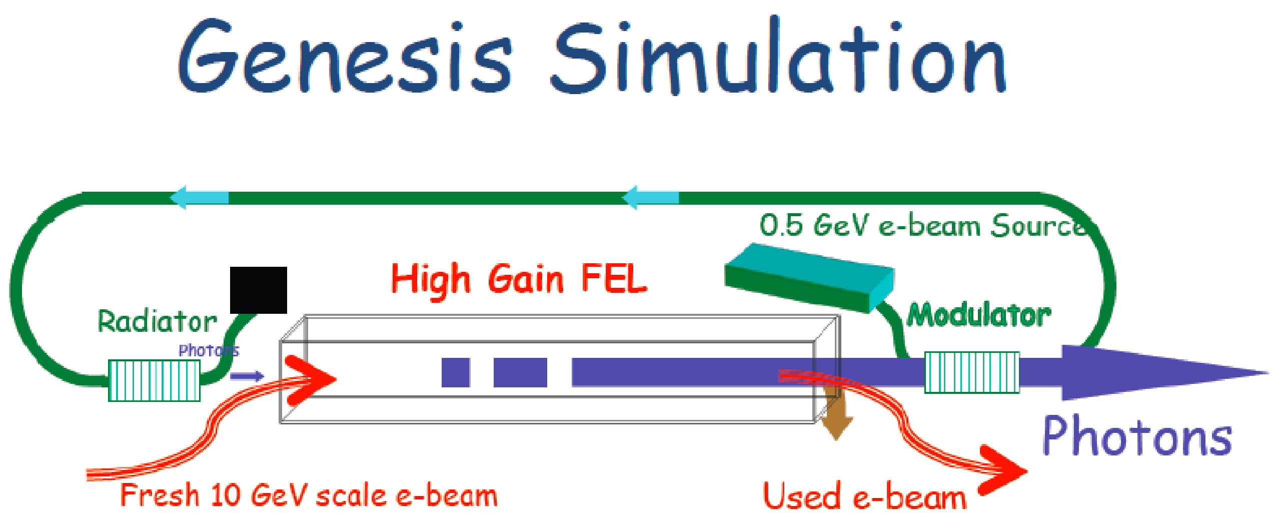
- FEL power builds up from noise to saturation in N passes inside an optical cavity. In the Nth pass, a fresh bunch of randomly distributed electrons interacts with the optical beam and develops microbunching with period = λ.
- At high intensity, the electrons rotate in phase space and absorb light near the end of the wiggler, and the large-signal gain is reduced compared to g_{ss} . Saturation occurs when gain is equal to total cavity losses.

But hard to find proper mirrors for hard X-ray!



OFFELO

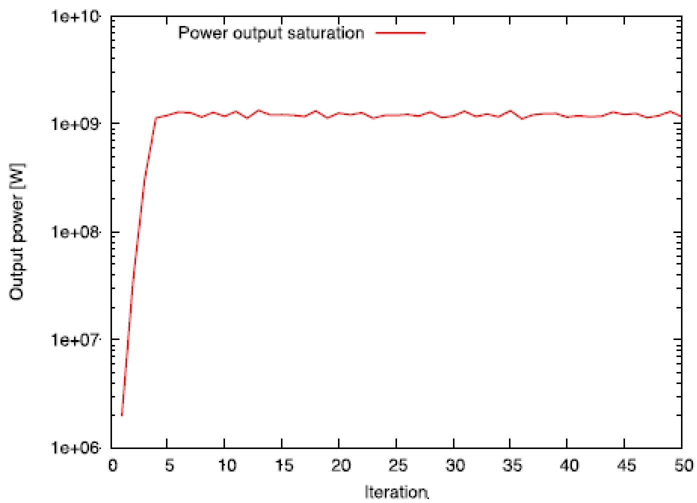
- OFFELO(Optics Free FEL Oscillator) is recently proposed to remove the limit on reflective mirrors.

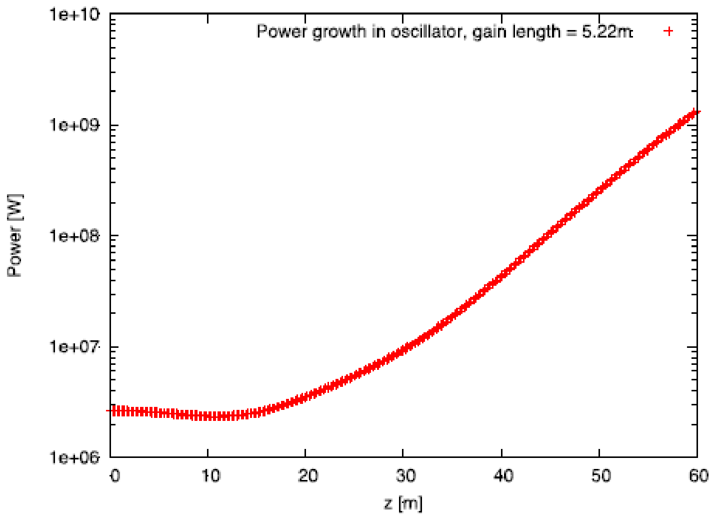


Courtesy of Yue Hao

Low energy beam is used as a feedback loop and carry the radiation information from modulator to radiator and act on the high energy beam again as a seed.

OFFELO performance





2 GENESIS input files are used and connected by a shell script to form an iteration and the peak power reaches saturation at 4th iteration with power exponential growth in the main oscillator.

OFFELO has the advantage as oscillator FEL with much narrower frequency spectrum for light than SASE FEL.

Conclusion and future plan

- A design of ultimate storage ring is undergoing and it can provide totally transverse coherent hard X-ray.
- Microwave instability will induce large energy spread growth thus SASE FEL is hard to achieve. OFFELO mechanism would be carried out in future simulations.
- More studies will be done on understanding how driving terms and tune shift with amplitude affect DA and further improvement of the storage ring's DA using GA is undergoing.

Thank you for your attention!